

# Non-linear Vibration Energy Harvesting in MEMS/NEMS

Y. Raviteja<sup>1</sup> and Manoj Pandey<sup>2</sup>

<sup>1,2</sup>Indian Institute of Technology-Madras, Chennai-600042, India  
E-mail: <sup>2</sup>mpandey@iitm.ac.in

---

**Abstract**—Energy harvesting has been the subject of active research in the last decade. In particular mechanical vibrations offer a significant amount of energy. Most of the vibration energy harvesters are based on the linear oscillators. The principal challenge in the linear oscillators is that they are well suited for stationary and narrowband excitation near their natural frequencies but are less efficient when the ambient vibrational energy is distributed over a wide spectrum. They may change in spectral density over time, and is dominant at very low frequencies. Nonlinear vibration energy harvesters are being explored to overcome these challenges and improve the efficiency. To date, a number of nonlinear energy harvesting studies have been conducted, mostly focusing on the monostable Duffing, impact, and bistable oscillator designs. The general dynamics of Bi-stable magnetic repulsion energy harvesters are discussed below. How does the energy harvesting changes when we apply nonlinearity is discussed below. The variation in Power output with the variation of coupling coefficient is studied. An equivalent Duffing oscillator equation to the Bi-stable magnetic repulsion energy harvesters is derived and its accuracy is compared with numerical simulation results.

**Index Terms:** MEMS, Energy harvester, Nonlinear dynamics.

## 1. INTRODUCTION

Energy harvesting has emerged as a prominent area and is growing at a rapid pace. There are wide range of applications for energy harvesting including embedded and implanted sensor nodes, refilling the batteries of large systems, self-powered sensors, powering unmanned vehicles, human-powered energy harvesting and smart systems. The most common sources for ambient energy are solar power, thermal gradients, acoustics and mechanical vibrations. In particular mechanical vibrations offer a significant amount of energy. Most of the vibration energy harvesters are based on the linear oscillators. The principal challenge in the linear oscillators is that they are well suited for stationary and narrowband excitation near their natural frequencies but are less efficient when the ambient vibrational energy is distributed over a wide spectrum. They may change in spectral density over time, and is dominant at very low frequencies. Nonlinear vibration energy harvesters are being explored to overcome these challenges and improve the efficiency. To date, a number of nonlinear energy harvesting studies have been conducted,

mostly focusing on the monostable Duffing, impact, and bistable oscillator designs. Monostable Duffing harvesters exhibit a broadening resonance effect dependent on the nonlinearity strength, device damping, and excitation amplitude, and thus can widen the unstable bandwidth of effective operation. Impact harvesters provide a mechanism for frequency up-conversion by using lower ambient vibration frequencies to impulsively excite otherwise linear harvesters so that they may ring down from much higher natural frequencies. Bistable duffing oscillator is discussed below.

## 2. BISTABLE DUFFING OSCILLATOR:

Bistable oscillators have a unique double-well restoring force potential, as depicted in Fig. 1 below. This provides for three distinct dynamic operating regimes depending on the input amplitude. Bistable devices may exhibit low-energy intrawell vibrations. In this case, the inertial mass oscillates around one of the stable equilibria with a small stroke per forcing period. Alternatively, the bistable oscillator may be excited to a degree so as to exhibit aperiodic or chaotic vibrations between wells. As the excitation amplitude is increased still further, the device may exhibit periodic interwell oscillations. The periodic interwell vibrations—alternatively, high energy orbits or snap-through—have been recognized as a means by which to dramatically improve energy harvesting performance. As the inertial mass must displace a greater distance from one stable state to the next, the requisite velocity of the mass is much greater than that for intrawell or chaotic vibrations. Since the electrical output of an energy harvester is dependent on the mass velocity, high-energy orbits substantially increase power per forcing cycle, which is preferable for external power storage circuits.

## 3. MODEL

The approach that will be pursued in this work is based on the exploitation of the dynamic properties of nonlinear oscillators, in particular bistable systems.

The Fig. below shows the nonlinear mechanism used here. The bistable principle is implemented using a cantilever beam

with a permanent magnet deposited onto the top surface. A fixed magnet is placed in front of the tip of beam at a distance  $\Delta$  which repels the cantilever tip.

The nonlinear system has been modelled considering a mass spring damper system with an additive nonlinear term described by a bistable potential energy function:

$$m\ddot{x} + c \frac{dx}{dt} + \frac{dU}{dx} + K_v V = F(t) \quad (1)$$

$$C_p \frac{dV}{dt} + \frac{V}{R} - K_c \frac{dx}{dt} = 0 \quad (2)$$

Where  $m$  = effective mass of the beam

$x$  = effective displacement of the beam

$c$  = effective damping coefficient of the beam

$t$  = time

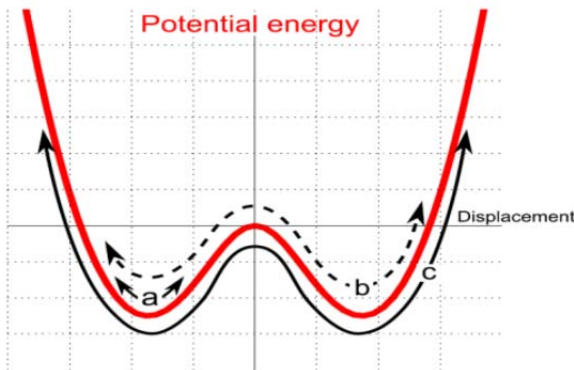
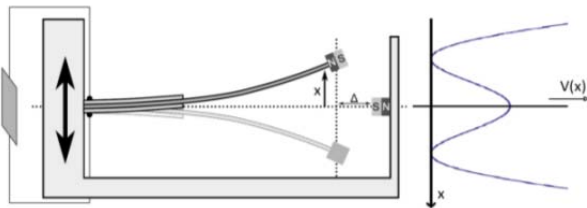
$U$  = potential energy of the beam

$F(t)$  = Force acting on the beam

$C_p$  = Capacitance of the piezo-electric material

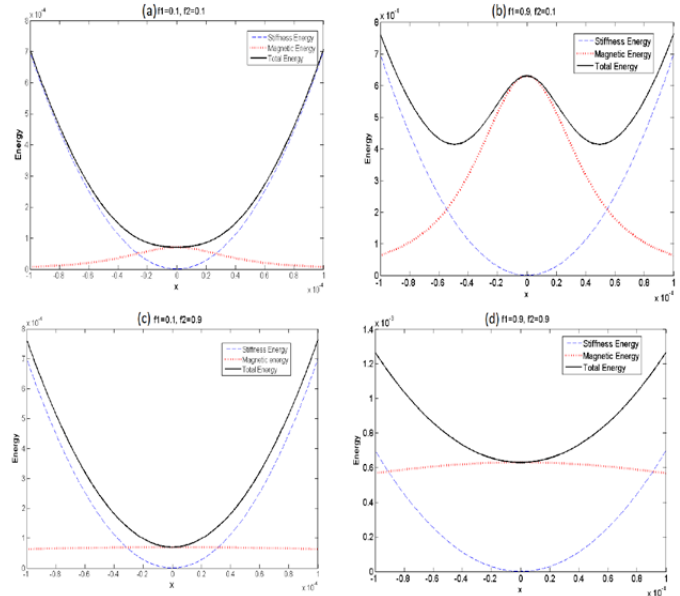
$V$  = Voltage across the load resistance  $R$  in the piezo-electric harvesting component

$=$  are the coupling coefficients  $v$   $K$   $c$   $K$



**Figure 1.** Double-well restoring force potential of a bistable oscillator showing example trajectories for (a) intrawell oscillations, (b) chaotic interwell vibrations and (c) interwell oscillations.

$U$  has two terms one from the stiffness of beam, other from the magnetic field.



**Fig. 2:** The variation of energy potential for different values of parameters  $f_1$  and  $f_2$ .

$$U = \frac{1}{2} kx^2 + (ax^2 + b\Delta^2)^{-3/2} \quad (3)$$

Where  $k$  = effective stiffness of the beam

$\Delta$  = distance between the tip of the beam and fixed magnet

$$a = d^2 \left( \frac{\mu_o}{2\pi} \frac{M^2}{d} \right)^{-2/3}$$

$$b = a / d^2$$

where  $d$  is a geometrical parameter depending on the point of measurement of  $x$  and tip of the beam.

We can replace the term with an equivalent damping and the mechanical equation becomes uncoupled second order differential with variable  $x$ .

$$m\ddot{x} + g \frac{dx}{dt} + kx - 3ax(ax^2 + b\Delta^2)^{-5/2} = F(t) \quad (5)$$

Power output can be computed as damping energy.

$$Power = \frac{1}{2} \frac{g}{t_2 - t_1} \int_{t_1}^{t_2} \left( \frac{dx}{dt} \right)^2 dt$$

$U$  has two terms one from the stiffness of beam, other from the magnetic field.

$$U = \frac{1}{2} kx^2 + (ax^2 + b\Delta^2)^{-3/2}$$

$$\text{Magnitude factor} = f1 = ((b\Delta^2)^{-3/2}) / (\frac{1}{2}k * x\_range^2)$$

$$\text{Sharpness factor} = f2 = (1 + \frac{d^2x\_range^2}{\Delta^2})$$

Where  $x\_range$  is the boundary value of domain  $x$ . The dependence of the shape of the potential function for different values of the magnitude and the sharpness factors is shown in Fig. 2.

After deciding the values of  $f1$  and  $f2$ , we can get the values of  $M$  and  $\Delta$  by using

$$\Delta = \frac{dx_r}{\sqrt{(f^{\frac{2}{3}} - 1)}} \quad M = \sqrt{\frac{2\pi d * f2 * kx_r^2 \Delta^3}{\mu}}$$

### 4. RESULTS

We solve (5) for different values of frequency ratios for time long enough so that steady state is reached and half the difference between the maximum and minimum displacements at steady state is noted as steady state Amplitude at that frequency ratio. This can be used to determine the power output as given in Fig. 3. The parameter values used are

- $m = .001\text{kg};$
- $c = 0.1\text{Nsm}^{-1};$
- $k = 700\text{Nm}^{-1};$
- $M = 0.0048\text{Am}^2;$
- $\Delta = 1.25663706e-6;$
- $xr = 1e-3;$
- $f1 = 0.1;$
- $f2 = 0.6;$
- $F = .4\cos(\omega t)\text{N};$
- Frequency ratio = 0.9;
- $R = 1\Omega;$
- $C = 100\text{nF}$

It is seen that there is improved energy extraction in the case of nonlinear system as opposed to the linear system.

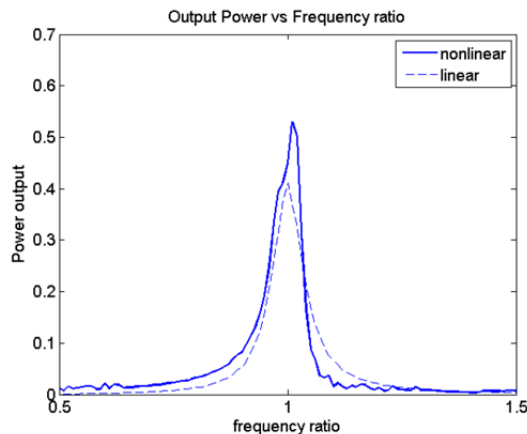


Fig. 3: Variation of power generation with input frequency.

Till frequency ratio=1.035, nonlinear case is giving more power than the linear case. For frequency ratio > 1.035, output power of nonlinear case is less than the linear case. The point of transition from intrawell vibrations to chaotic vibrations is closer to the point of intersection of nonlinear and linear power graph. For the intrawell oscillations nonlinear oscillations are giving more power than the linear oscillations. Chaotic oscillations are giving lesser power than the linear oscillations.

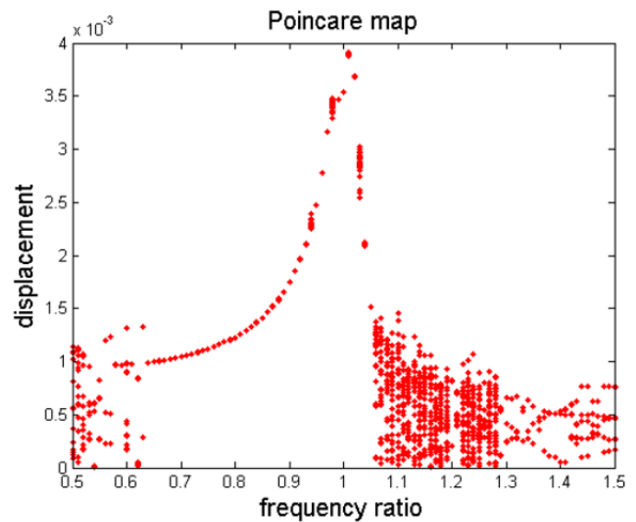
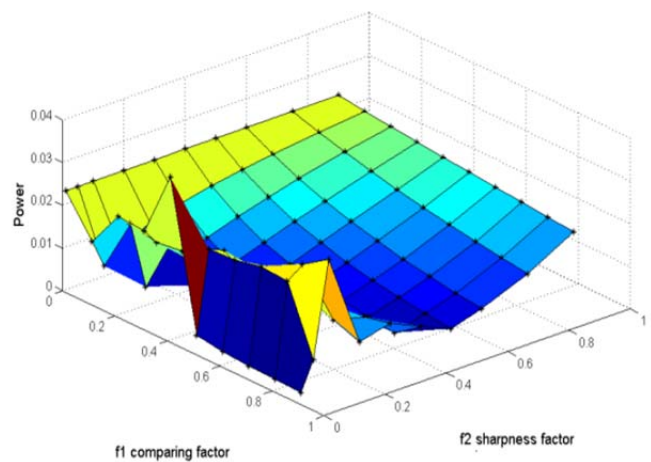
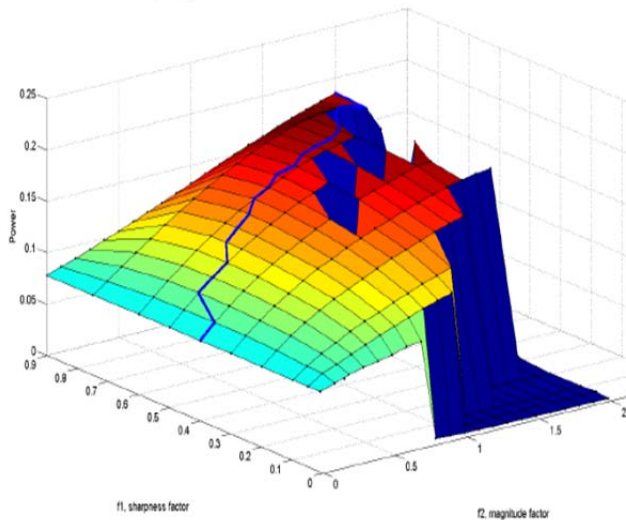


Fig. 4: Bifurcation diagram for the system with varying frequency, showing chaotic regions.

### Variation of power with $f1$ and $f2$

We varied  $f1$  and  $f2$  to see what values of  $f1$  and  $f2$  are giving the maximum power in the case of intrawell oscillations (as shown in fig. 3.4 frequency ratio = 0.9 corresponds to intrawell oscillations) and chaotic oscillations.





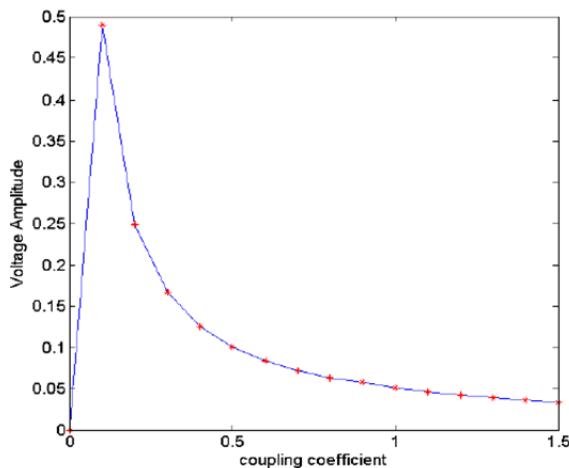
**Fig. 5: Variation of power with parameters  $f_1$  and  $f_2$ . For frequency ratio of a) 1.26 b) 0.96**

Next we look at the variation of power with the magnitude and the sharpness factors.  $f_1 = 0$ , i.e. Magnetic Energy = 0

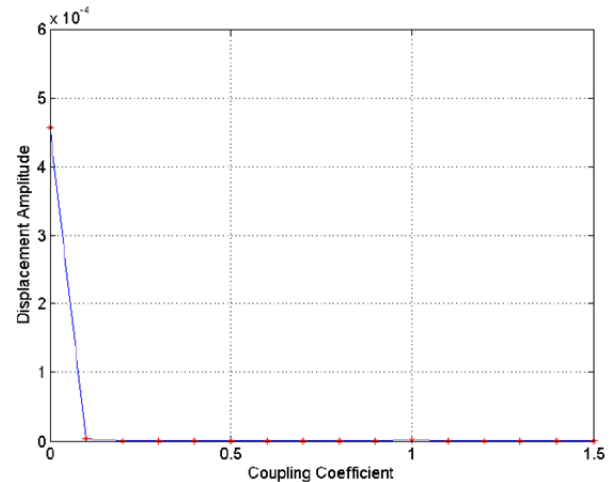
represents linear system. For all values of  $f_1, f_2$ , Power output is smaller than that of linear model (as shown in Fig. 3.4 frequency ratio = 1.26 corresponds to chaotic oscillations).

From Fig. 3.5, as  $f_2$  is increasing every system turned into chaotic oscillations.

And at chaotic oscillations the power output is lower than the linear system. Output of linear oscillator can be seen from Fig. 3.5 at  $f_2 = 0$ . As  $f_2$  is increasing Power is increasing (Fig. 3.6) as long as it does not encounter a chaotic oscillation. And after certain  $f_2$ , intrawell oscillations are turning into chaotic oscillation for all  $f_1$ . At  $f_2 = 0$ , Power is equal for all  $f_1$ . For all other  $f_2$ , maximum Power increases with  $f_1$  reaches a maximum and then decreases (Fig. 3.7).



**Fig. 6: Variation of Voltage amplitude with the coupling coefficient.**



**Fig. 7: Variation of Displacement amplitude with the coupling coefficient.**

The variation of  $f_1$  corresponding to maximum Power at a constant  $f_2$  with  $f_2$  is shown in Fig. 3.8. Blue line on the surface connects the points that are maximum for a constant  $f_2$ . For all the values of  $f_1$  and  $f_2$ , chaotic oscillations are giving Power less than the

Linear system. For a given value of  $f_1$ , Power is increasing with  $f_2$  reaching a maximum and then is decreasing. Higher the value of  $f_1$ , Higher is the Power output for all values of  $f_2$ , if the oscillations are intrawell oscillations. But for every  $f_2$ , there is an upper limit of  $f_1$  for the oscillation to be intrawell. Optimum value of Power = 0.20W Power for  $f_2 = 0$ , i.e. linear system = 0.078W.

**Variation of power with electric coupling coefficient**

Fig. 6 shows variation with the coupling coefficients. For coupling coefficient,  $K_c = 0$ , output Voltage = 0. But for the immediate next value of  $K_c$  ( $K_c = 0.1$ ), Voltage is reaching maximum and then is decreasing. Hence it is better to have coupling coefficient as small as possible.

**5. CONCLUSION**

In this work, bi-stable magnetic repulsion energy harvester was analyzed for the optimum energy harvesting to some extent by using numerical simulation method. Nonlinear bi-stable magnetic repulsion harvester is giving more output power than linear harvester only when the nonlinear system is oscillating in intrawell regime. But in chaotic regime the output power is lesser than the linear oscillator. Variation of Power with  $f_1$  and  $f_2$  is shown. There is lower limit for  $f_2$  so that the oscillations are in intrawell regime. Power is maximum at this  $f_2$ . For  $f_2$  between 0 and limit of  $f_2$  for intrawell regime,  $f_1$  corresponding to maximum Power is ranging between 0.4 and 0.6. By selecting optimum values of  $f_1$  and  $f_2$ , the increase in energy harvesting can be about 100% more than the linear system. The lower the Coupling

---

coefficient, higher the output energy. Equivalent Duffing oscillator and Steady state solution was derived for the magnetic repulsion harvester equation, which are working well for high values of  $\Delta$ .

## REFERENCES

- [1] R L Harne and KW Wang, 2013, A review of the recent research on vibration energy harvesting via bistable systems IOP Science.
- [2] And' o B, Baglio S, Trigona C, Dumas N, Latorr L and Nouet P, 2010, Nonlinear mechanism in MEMS devices for energy harvesting applications J. Micromech. Microeng.
- [3] Cottone F, Vocca H and Gammaitoni L, 2009, Nonlinear energy harvesting Phys. Rev. Lett.
- [4] Karami M A and Inman D J, 20011, Equivalent damping and frequency change for linear and nonlinear hybrid vibrational energy harvesting systems J. Sound Vib.